Asian Journal of Control, Vol. 17, No. 2, pp. 582–591, March 2015
Published online 9 May 2014 in Wiley Online Library (wileyonlinelibrary.com) DOI: 10.1002/asjc.893

NON-RULE BASED FUZZY APPROACH FOR ADAPTIVE CONTROL DESIGN OF NONLINEAR SYSTEMS

Yinhe Wang, Liang Luo, Branko Novakovic, and Josip Kasac

ABSTRACT

A novel adaptive control approach is presented using extended fuzzy logic systems without any rules. First, the extended fuzzy logic systems without any rules are used to approximate the uncertainties. Then the sliding mode controllers via the proposed extended fuzzy logic systems without any rules are proposed for uniformly ultimately bounded (UUB) nonlinear systems. The adaptive laws are used for estimating the approximation accuracies of fuzzy logic systems without any rules, Lipschitz constants of uncertain functions and scalar factor, respectively, which are not directly to estimate the coefficients of basis functions. Finally, a compared simulation example is utilized to demonstrate the effectiveness of the approach proposed in this paper.

Key Words: Fuzzy logic systems without any rules, adaptive control, UUB.

I. INTRODUCTION

Fuzzy control design is a fundamental method in the control theory [1–12]. However, in the previously described conventional fuzzy adaptive control (FAC) methods [1–3,5–7], the Mamdani fuzzy rules or Takagi–Sugeno (T–S) fuzzy rules are employed, and thus two substantial drawbacks are shown. The first is that the exponential growth in rules accompanied by the number of variables increases, because the input space of the fuzzy logic system (FLS) is generated via grid-partition [13,14]. A few works [15–17] present a new, nonconventional analytic method for synthesis of the fuzzy control by using fuzzy logic systems without any rules (FWR). However, the output of FWR can not be rewritten as a linear combination of fuzzy basis functions. Hence, the FWR is unsuitably employed in the conventional adaptive fuzzy control algorithms [1–3,5–11].

The second drawback is that FAC easily leads to complex adaptation mechanisms. In order to solve this problem, more recently several new adaptive fuzzy control schemes have been proposed in [5–7,18–20] for nonlinear systems with triangular structure. The general idea of these methods is to use the norm of the ideal weighting vector in fuzzy logic systems as the estimated parameter, instead of the elements of weighting vector. However, each virtual controller needs to induce new state variable. In addition, the above methods [5–7,18–20] can be applied only to the FLS with if-then rules, due to the outputs of FLS can be written as linear combination of fuzzy basis functions. This limits the applications of the other types of fuzzy logic system such as the FWR in [15–17].

In order to overcome the above two shortcomings, the FWR are used to approximate the uncertainties of the controlled systems. In this paper, in order to put the FWR together with the usual adaptive method, the scalar and saturator with adjustable parameters are employed and are serially connected with the input port of the FWR to form the extended FWR. By using the extended FWR, the sliding mode controllers via the parameter adaptive laws are proposed for a class of nonlinear uncertain systems such that the states of the controlled systems are uniformly ultimately bounded (UUB). The parameter adaptive laws in this paper are designed to adjust approximate accuracies of extended FWR, scalar factor, and Lipschitz constants of uncertainties, respectively, rather than to estimate the coefficients in the linear combination of fuzzy basis functions. This implies that the two processes of constructing the FWR and designing adaptive laws may be separated. This will helpfully serve in the process of choosing of the suitable FWR for obtaining better approximate accuracies. The above idea of adaptive fuzzy control is involved in [21]. However, in [21] the FWR are not employed and the unknown functions are request to be continuous homogeneous functions, which limit the category of unknown functions. In this paper, the FWR are employed

Manuscript received April 14, 2013; revised October 10, 2013; accepted January 19, 2014.

Yinhe Wang (e-mail: yinhewang@sina.com) and Liang Luo (corresponding author, e-mail: liangluo825@163.com) are with the School of Automation, Guangdong University of Technology, Guangzhou, China.

Liang Luo is also with the College of Mathematics and Information Science, Shaoguan University, Shaoguan, Guangdong, China.

Branko Novakovic (e-mail: branko.novakovic@fsb.hr) and Josip Kasac (e-mail: josip.kasac@fsb.hr) are with the Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, HR-10000 Zagreb, Croatia.

This work is supported by the National Natural Science Foundation of China (No. 61273219, No. 61305036), National Natural Science Foundation of Guangdong (No. S2013010015768, No. S2013040013503), and Specialized Research Fund for the Doctoral Program of Higher Education of China (20134420110003).

and the unknown functions just satisfy Lipschitz conditions instead of homogeneous condition. The proposed method in this paper is a unified adaptive law design scheme suited to the FWR.

II. FUZZY SYSTEM WITHOUT FUZZY RULE BASE

In this section, the FWR in [15–17] is introduced. These fuzzy sets are defined only for the normalized input variables with the following membership functions

$$s_i^j(x_j)=\begin{cases} 0, & x_j=z_{i0}^j \ or \ z_{ie}^j \\ \dfrac{1-\cos\left(\dfrac{2\pi\varepsilon_i^j(x_j-x_{ic}^j+T_i^j/2)}{(\varepsilon_i^j-1)T_i^j}\right)}{2}, & z_{i0}^j<x_j<z_{ia}^j \\ \dfrac{1-\cos\left(\dfrac{2\pi\varepsilon_i^j(x_{ic}^j+T_i^j/2-x_j)}{(\varepsilon_i^j-1)T_i^j}\right)}{2}, & z_{ib}^j<x_j<z_{ie}^j \\ 1, & z_{ia}^j\le x_j\le z_{ib}^j, \end{cases} \tag{1}$$
$$i=1,\ldots,n_j,\quad j=1,\ldots,m.$$

where x_j are input variables, m is the number of input variables and n_j is the number of fuzzy sets belonging to the jth input variables. The parameter z_{i0}^j denotes the beginning and z_{ie}^j the end of the ith fuzzy set on the abscissa axis. The centre of ith fuzzy set is denoted by x_{ic}^j, while ith fuzzy set basis is $T_i^j=z_{ie}^j-z_{i0}^j$. The parameters ε_i^j are defined by the equation: $\varepsilon_i^j=\dfrac{z_{ie}^j-z_{i0}^j}{z_{ib}^j-z_{ia}^j}, \varepsilon_i^j>1.$

The normalized input variable $\underline{x}j = xj/|xj\max|$, x_j is jth input variable, $j = 1, \ldots, m$; $xj\max$ is the maximum value of x_j.

In [15–17], a special distribution of input fuzzy sets is used. This has been done by the following modification of the fuzzy set shape from (1):

$$\underline{s}_i^j(\underline{x}_j)=\begin{cases} 1, & \underline{x}_{ia}^j\le\underline{x}_j\le\underline{x}_{ib}^j \\ \dfrac{s_i^j(\underline{x}_j)}{\exp\left(\beta_j\varepsilon_i^j|\underline{x}_j|\right)}, & \underline{x}_j\le\underline{x}_{ia}^j, or\ \underline{x}_j\ge\underline{x}_{ib}^j\ or\ \underline{x}_{ib}^j\ge\underline{x}_{ia}^j, \\ i=1,\cdots,n_j,\ j=1,\cdots,m \end{cases} \tag{2}$$

where ε_i^j and $\beta_j\ge 0$ are free adaptation parameters.

By using (2) and sum/product inference operators, the new activation function ω_j of the jth output fuzzy set can be proposed as

$$\omega_j(\underline{x}_j)=\sum_{i=1}^{n_j}\underline{s}_i^j(\underline{x}_j),\ j=1,\cdots,m. \tag{3}$$

The activation function ω_j denotes the grade of membership of input $\underline{x}_j$ to all of the input fuzzy sets.

$$\underline{y}_{jc}(\underline{x}_j)=\left(1-\frac{\omega_j}{n_j}\right)\operatorname{sgn}(\underline{x}_j),\ j=1,\cdots,m \tag{4}$$

where $\underline{y}_{jc}(\underline{x}_j)$ denotes the normalized position of center of the corresponding output fuzzy set.

Since the input variables are normalized, it requires a determination of a gain K_{cj} of output set center position. The gain K_{cj} is proposed as $K_{cj}=U_mF_j\left(1+|\underline{x}_j|^{a_j}\right)$, where $U_m>0$, $F_j>0$ and $a_j>0$ are the maximum value of the position of center of the corresponding output fuzzy set and free parameters, respectively.

By using the gain K_{cj}, the output fuzzy set center position is obtained as

$$y_{cj}=U_mF_j\left(1+|\underline{x}_j|^{a_j}\right)\underline{y}_{cj}(\underline{x}_j),\ j=1,\cdots,m \tag{5}$$

Finally, the proposed output of the FWR in [15–17] has been described below:

$$E(x_1,\cdots,x_m)=\frac{\sum_{j=1}^{m}\dfrac{\omega_j(\underline{x}_j)y_{jc}(\underline{x}_j)T_j(\varepsilon_i^j+1)}{2\varepsilon_i^j}}{\sum_{j=1}^{m}\dfrac{\omega_j(\underline{x}_j)T_j(\varepsilon_i^j+1)}{2\varepsilon_i^j}} \tag{6}$$

where the constant T_j is jth fuzzy set basis, ε_i^j are adjustable parameters. More details about the FWR are available in [15–17].

Remark 1. (i) In this new approach the number of fuzzy system input variables and the number of input fuzzy sets are not limited. (ii) It can be seen from (6) that the output of FWR may be not in the usual form (linear constant combination of the fuzzy basis functions) In this paper, we propose an adaptive control scheme for a class of nonlinear uncertain systems by using scalar, saturator and the output (6) of the FWR.

III. PRELIMINARIES AND THE FWR WITH SCALAR AND SATURATOR

Definition 1. The mapping: $\phi{:}x=\rho x$ is called a scalar, noting that $\phi(x)=\rho x$, where the real ρ is called scalar factor, $x=(x_1\ \cdots\ x_n)^T\in R^n$.

Definition 2 [22]**.** The mapping: $sat{:}x\mapsto sat(x)$ is called a (vector) saturator, and the saturator function is defined as follows:

$$sat(x)=(sat(x_1)\ \cdots\ sat(x_n))^T,$$

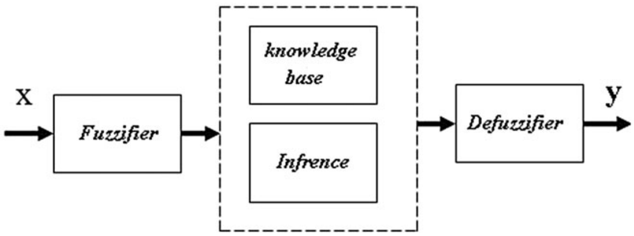

Fig. 1. Basic configuration of the fuzzy system without rules (FWR).

$$sat(x_i)=\begin{cases}-\alpha_i,\ x_i<-\alpha_i\\ x_i, |x_i|\le\alpha_i \quad ,i=1,\cdots,n\\ \alpha_i,\ x_i>\alpha_i\end{cases}$$

where α_i is positive real numbers, and α is the minimum saturated degree of the whole α_i, that is $\alpha=\min\limits_{1\le i\le n}\{\alpha_i\}$.

Remark 2. (i) If $\alpha_i = 1(i = 1, 2, \ldots, n)$, then $sat(x)$ in Definition 2 is called a normalized unit saturator [22]; (ii) It is easy to verify that $sat(x) = x$ for $\|x\| \le \alpha$, where $\|x\|$ is the Euclidean norm.

The FWR is shown in Fig. 1 with the output (6) abbreviated as

$$y=FWR(x) \tag{7}$$

where $x = (x_1 \ \ldots \ x_m)^T$ and the knowledge base is not represented in the form of the fuzzy if-then rules. From input interface to output interface, an intuitive reasoning is introduced in [15–17] to mapping the input to the center position of output fuzzy set by using the activation function (3) and the output fuzzy set center position function (5).

Now, a scalar and saturator are in series with the input port of FWR in Fig. 1, to form the extended FWR (EFWR).

Here the scalar factor of the input port in Fig. 2 is $\frac{1}{\rho}$, and α is the minimum saturated degree of the saturator in the input port.

From (7) and Fig. 2, the output of the extended FWR is given by

$$\bar{y}=EFWR\left(sat\left(\frac{x}{\rho}\right)\right). \tag{8}$$

Based on the Definition 2, and provided that the inequality $\left\|\frac{x}{\rho}\right\|\le\alpha$ is satisfied, the following property holds: $\bar{y}=EFWR\left(\frac{x}{\rho}\right)$, where $\bar{y}$ denotes the output of the extended FWR.

Lemma 1. Consider a continuous function $\psi(z)$ in a closed bounded set Ω, which satisfies L-Lipschitz conditions. If for real $E > 0$ (approximation accuracy), there exists an FWR such that the following approximate result is true on the set $\bar{U}=\{z\|\|z\|\le\alpha, z\in R^n\}\subseteq\Omega$,

$$\sup_{\|z\|\le\alpha}|\psi(z)-FWR(z)|\le E \tag{9}$$

then the following approximate property holds on the set $\bar{U}$:

$$\sup_{\|z\|\le|\rho|\alpha}\left|\psi(z)-EFWR\left(\frac{z}{\rho}\right)\right|\le L\|z\|\left|1-\frac{1}{\rho}\right|+E \tag{10}$$

where the output of the extended FWR (in Fig. 2) is described with (8).

IV. SYSTEM DESCRIPTION AND SOME ASSUMPTIONS

In this paper, we consider the single input single output (SISO) nonlinear system characterized by

$$x^{(n)}=f(z)+gu \tag{11}$$

whe $u\in R$ and $z=(x\ \dot{x}\cdots x^{(n-1)})^{\mathrm{T}}\in\tilde{U}\subseteq R^n$ are control input and state vector, respectively. Let $\tilde{U}$ be a compact set; $f(z)$ is an unknown continuous function and g is an unknown constant gain.

In that case the system (11) can be rewritten as

$$\dot{z}=Az+B[f(z)+gu] \tag{12}$$

where $A=\begin{pmatrix}O & I_{n-1}\\ 0 & O^T\end{pmatrix}$, $B=(O^T\ 1)^T$, O denotes n − 1 column vector with all elements 0, I_{n-1} denotes n − 1 order identity matrix.

Note that the pair (A, B) is completely controllable. For given a positive definite matrix Q and vector$K$, one should solve the following equation:

$$(A+BK)^TP+P(A+BK)=-Q \tag{13}$$

for $P > 0$. Such solution of P exists since $A + BK$ is asymptotically stable.

Assumption 1. For the compact set $\tilde{U}$, there exist two known positive constants $g_{\min}$, $g_{\max}$ such that $0 < g_{\min} \le g \le g_{\max}$.

Assumption 2. (i) Consider the system (11), and assume that the state set $\{z\|z\|\le\alpha\}\subseteq\tilde{U}$ can be defined by choosing the parameter α. (ii) If Assumption 1 is satisfied, there exists an unknown positive real number E_1 and the FWR_1 such that $\sup\limits_{z\in\tilde{U}}|\sigma_1(z)-FWR_1(z)|\le E_1$, where $\sigma_1(z)=\frac{g_{\max}}{g}f(z)$. (iii) There exists another FWR_2 and an unknown positive real

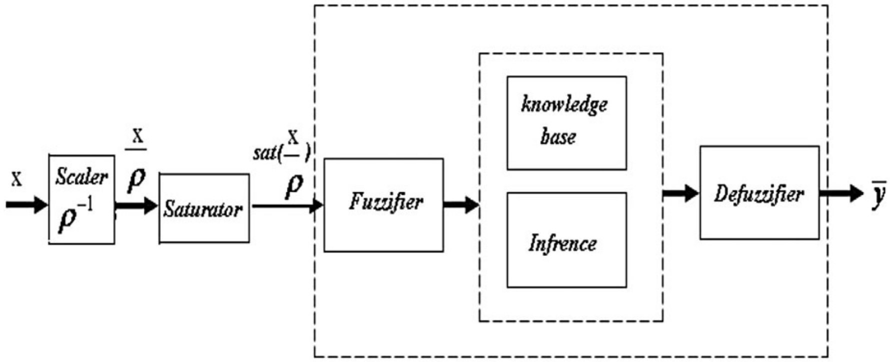

Fig. 2. The extended FWR (EFWR) with scalar and saturator.

number E_2 satisfying $\sup_{z\in\bar{U}}|\sigma_2(z) - FWR_2(z)| \leqslant E_2$, where $\sigma_2(z) = -\frac{g_{\max}}{g}Kz$ and K is a matrix such that $A + BK$ is Hurwitz stable.

V. ADAPTIVE FUZZY CONTROL DESIGN BASED ON FWR

In application, E_i, L_i, $i = 1, 2$ are unknown. Let $\hat{E}_i, \hat{L}_i$ denote the estimation of E_i, L_i, and $\tilde{E}_i = \hat{E}_i - E_i$, $\tilde{L}_i = \hat{L}_i - L_i$ the estimate error, respectively. For simplicity, the following notations are used.

$$E = (E_1 \quad E_2)^{\mathrm{T}}, \tilde{E} = (\tilde{E}_1 \quad \tilde{E}_2)^{\mathrm{T}}, \hat{E} = (\hat{E}_1 \quad \hat{E}_2)^{\mathrm{T}} \tag{14a}$$

$$L = (L_1 \quad L_2)^{\mathrm{T}}, \tilde{L} = (\tilde{L}_1 \quad \tilde{L}_2)^{\mathrm{T}}, \hat{L} = (\hat{L}_1 \quad \hat{L}_2)^{\mathrm{T}} \tag{14b}$$

Consider the following extended closed-loop system (ECS).

$$\dot{z} = Az + B[f(z) + gu] \tag{15a}$$

$$\dot{\rho} = \eta(z, \rho, \hat{E}, \hat{L}) \tag{15b}$$

$$\dot{\hat{E}} = \vartheta_1(z, \rho, \hat{E}) \tag{15c}$$

$$\dot{\hat{L}} = \vartheta_2(z, \rho, \hat{L}) \tag{15d}$$

$$u = u(z, \rho) \tag{15e}$$

where the state vector of the ECS (15) is $\mathbb{Z} = (z^{\mathrm{T}}, \rho, \hat{E}^{\mathrm{T}}, \hat{L}^{\mathrm{T}})^{\mathrm{T}}$. The mappings $\eta(*)$ (the updated law of the parameter ρ), $\vartheta_1(*)$, $\vartheta_2(*)$ (the adaptive law of estimate value of E and L, respectively) and the controller $u = u(z, \rho)$ will be designed according to the following control goal.

Control goal. Design the controller (15e), updated law (15b) and adaptive laws (15c) and (15d) such that the state vector $\mathbb{Z} = (z^{\mathrm{T}}, \rho, \hat{E}^{\mathrm{T}}, \hat{L})^{\mathrm{T}}$ is uniformly ultimately bounded (UUB).

Case 1. $\|z\| > |\rho|\alpha$

In this case, we adopt open-loop control, that is $u = 0$, and use the FWR_1 to approximate the nonlinear function $\sigma_1(z)$. Meanwhile, the updated law of $\rho = \rho(t)$ is proposed as follows:

$$\dot{\rho} = \frac{1}{2\rho\alpha^2}\left\{\lambda + 2\sqrt{n-1}\|z\|^2 + 2\|z\| \cdot \left(\hat{E}_1 + |EFWR_1|\right)\right. \tag{16}$$

where λ is an adjustable positive constant.

The adaptive laws of the estimated parameter vector are proposed as follows:

$$\dot{\hat{E}}_1 = 2\beta_1\|x\|, \dot{\hat{E}}_2 = 0, \dot{\hat{L}}_1 = 0, \dot{\hat{L}}_2 = 0, \tag{17}$$

with β_1 being a positive design constant.

We use the following Lemma 2 in order to prove that the state $\mathbb{Z} = (z^{\mathrm{T}}, \rho, \hat{E}^{\mathrm{T}}, \hat{L}^{\mathrm{T}})^T$ of the ECS (15) can reach $\mathrm{D} = \{\mathbb{Z}|\ \|z\| \leq |\rho|\alpha\}$ in finite time.

Lemma 2. Consider the ECS (15). If Assumptions 1 and 2, and the condition $\|z\| > |\rho|\alpha$ are true, then the above controller $u = 0$ and the updated laws, described by (16) and (17), can be ensured to force the state $\mathbb{Z} = (z^{\mathrm{T}}, \rho, \hat{E}^{\mathrm{T}}, \hat{L}^{\mathrm{T}})^T$ of the ECS (11) to reach the compact set $D = \{\mathbb{Z}|\ \|z\| \leq |\rho|\alpha\}$ in finite time.

Proof. See Appendix A.

Remark 3. (i) In the open-loop case, the updated law (16) ensures that the SSs can go into the effective range of the saturator. (ii) FWR_1 is used to approximate the unknown function $f(z)$ and to obtain the available information of their upper boundary.

Case 2. $\|z\| \leq |\rho|\alpha$

In this case, the two EFWR_i, $i = 1, 2$, are employed to synthesize the controller $u = u_1 + u_2$, where

$$u_1 = -\frac{1}{g_{\max}} EFWR_1\left(\frac{z}{\rho}\right) \tag{18a}$$

$$u_2 = -\frac{1}{g_{\max}} EFWR_2\left(\frac{z}{\rho}\right) \tag{18b}$$

The updated law of ρ is proposed as follows:

$$\begin{aligned}\dot{\rho} = &-\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}\rho - 2\gamma\rho\alpha^2\left|1-\frac{1}{\rho}\right|\|PB\|(\hat{L}_1+\hat{L}_2)\\ &-2\gamma\widetilde{sign}(\rho)\|PB\|\alpha(\hat{E}_1+\hat{E}_2)\end{aligned} \tag{19}$$

where γ is a positive design constant, and $\widetilde{sign}(\rho) = \begin{cases}1, & \rho > 0\\ -1, & \rho < 0\end{cases}$.

The related adaptive laws are proposed as follows:

$$\dot{\hat{E}}_1 = -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\hat{E}_1 + 2\delta_1|\rho|\alpha\|PB\| \tag{20a}$$

$$\dot{\hat{E}}_2 = -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\hat{E}_2 + 2\delta_1|\rho|\alpha\|PB\| \tag{20b}$$

$$\dot{\hat{L}}_1 = -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\hat{L}_1 + 2\delta_2\rho^2\alpha^2\left|1-\frac{1}{\rho}\right|\|PB\| \tag{20c}$$

$$\dot{\hat{L}}_2 = -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\hat{L}_2 + 2\delta_2\rho^2\alpha^2\left|1-\frac{1}{\rho}\right|\|PB\| \tag{20d}$$

where δ_1, δ_2 are positive design constants.

Lemma 3. Consider the ECS (15). If Assumptions 1 and 2 and the condition $\|z\| \le |\rho|\alpha$ are satisfied, then Controller (18), Updated law (19) and Adaptive law (20) ensure that the state $\mathbb{Z} = (z^{\mathrm{T}}, \rho, \hat{E}^{\mathrm{T}}, \hat{L}^{\mathrm{T}})^T$ is uniformly ultimately bounded (UUB).

Proof. See Appendix B.

Remark 4. (i) The controller u_1 consists of extended FWR_1 in form of Fig. 2, which serves against the affection of the unknown function in $f(z)$. The controller u_2 is in switched form to overcome the uncertainties in $\sigma_2(z)$. (ii) The updated law (19) is not directly connected with the state z of the system (11) but is connected with the estimate values of approximate accuracies of the FWR_i, $i = 1,2$.

Theorem 1. If Assumptions 1 and 2 are satisfied, then the state $\mathbb{Z} = (z^{\mathrm{T}}, \rho, \hat{E}^{\mathrm{T}}, \hat{L}^{\mathrm{T}})^T$ of the closed-loop system (15), associated with the control law and the adaptive laws in Case 1 and 2, are uniformly ultimately bounded (UUB).

Remark 5. (i) Adaptive laws have nothing to do with fuzzy basis functions. From the expression (19), we clearly see that the adaptive law in this paper is not directly related to the fuzzy basis functions. The same conclusion can be obtained in the expressions (16). In the conventional adaptive fuzzy methods, the adaptive laws are used to adjust the parameters with respect to the fuzzy basis functions. In this sense, one basis function needs to have one adaptive law that suffers from combinatorial rule explosion. (ii) The FWR can be employed in this approach. In [7–11], usually the fuzzy approximators have been used where the structure of outputs is a linear combination of fuzzy basis functions (such as Mamdani or TS type) in order to approximate nonlinear terms. These approximators can be easy used for adaptive technique in order to adjust the coefficients of the fuzzy basis function. But at the same time it brings restriction to us: when the approximation's structure of output is not in a linear combination of fuzzy basis functions (such as FWR), the previous methods [7–11] are ineffective. The approach proposed in this paper is effective in avoiding this problem.

VI. SIMULATION

In this section, we apply the proposed controller in this paper to an inverted pendulum system under different initial conditions. Consider the following second-order model of the inverted pendulum system:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f(z) + gu(t) + d(t)\end{aligned} \tag{21}$$

where $z = (x, \dot{x})^{\mathrm{T}}$, $x_1 = \theta(rad)$ and $x_2 = \dot{\theta}(rad/s)$. $u(N)$ is the applied force (control). Further, $d(t) = \sin(\pi t)\cos(\pi t)$ is the disturbance. The smooth functions $f(z)$ and g are unknown for synthesizing the controller in this paper. Assume that $f(0) = 0$, $1 \le g \le 2$.

We choose the design parameters as: $\beta_1 = \delta_1 = \delta_2 = 0.0001$, $\gamma = 1$, $\lambda = 1$, the minimum saturated degree $\alpha = 1$. $\rho(0) = 6$, $\hat{E}_1(0) = 0.4$, $\hat{E}_2(0) = 0.6$, $\hat{L}_1(0) = 0.8$, $\hat{L}_2(0) = 0.5$.

Based on the design ideas in this paper, we will need to construct two FWR for approximating the following nonlinear functions.

For the first nonlinear function $\sigma_1 = -g_{\max}g^{-1}K_z$, where $K = (-30\ -40)$, by using the methods in [15–17], we choose the parameters of FWR shown in Tables I–III.

For the second nonlinear function $\sigma_2 = f(z)$, the most simple form of the analytic function is considered (more details about the method may be seen in literature [15]).

$$\omega_{ji}(x_{ji}) = S_{ji}(x_{ji}) = \gamma_{ji} + \overline{\gamma}_{ji}\exp\left(-\alpha_{ji}x_j^2 - \beta_{ji}|x_{ji}|\right) \tag{22a}$$

Table I. Basis parameters of input fuzzy sets.

Fuzzy set NO.	Adaptable parameter ε	Base of fuzzy set	Fuzzy set center
1	2	2	0
2	2	2	0
3	2	2	0

Table II. Basis parameters of activation function.

Input variable NO.	X1	X2
Xmax	100	100
Parameter β	0.25	0.01

Table III. Basis parameters of output fuzzy sets.

Output fuzzy set no.	1	2
Parameterα	0.05	0.02
Gain F	0.06	0.01
Gain Um	10	10
ParameterT_0	10	8
Parameterε_0	20	30

Table IV. Basis parameters of out fuzzy sets.

Output fuzzy set NO.	1	2
Gain Kc	100000	300
Parameterβ	100000	1
Parameterγ	0.5	0.5
Parameter$I$	0.01	1

$$yc_{ji} = kc_{ji}(1 - \exp(-\beta_{ji}|x_{ji}|))sign(x_{ji}) \tag{22b}$$

where $j = 1, 2$, $i = 1, \ldots, N_j$, $N_j = 1$, $\alpha_{ji} = 0$.

The following two cases are considered in simulation.

Condition 1. The smaller initial condition state $x_1(0) = 0.19 \approx 11°$.

Fig. 3 shows the time responses of the angle in [5] (noted as CB) and [2] (noted as FLS) and the method proposed in this paper (noted as FWR). Fig. 4 is a locally-enlarged picture of Fig. 3. It is seen clearly that the controller in [2] fails to converge, while the controller in [5] and the controller in this paper still converges. Additionally, the controller in this paper has a shorter response time. Fig. 5 shows the updated law, and Fig. 6 shows the estimate value of approximate accuracies for the two FWR and Lipschitz constant. From Fig. 5 and Fig. 6, it can also be observed that the proposed updated law, approximate accuracies and Lipschitz constant in this paper quickly converge to zero.

Condition 2. The same simulation as Example 1 is performed except that the large initial condition state $x_1(0) = 1 \approx 57°$.

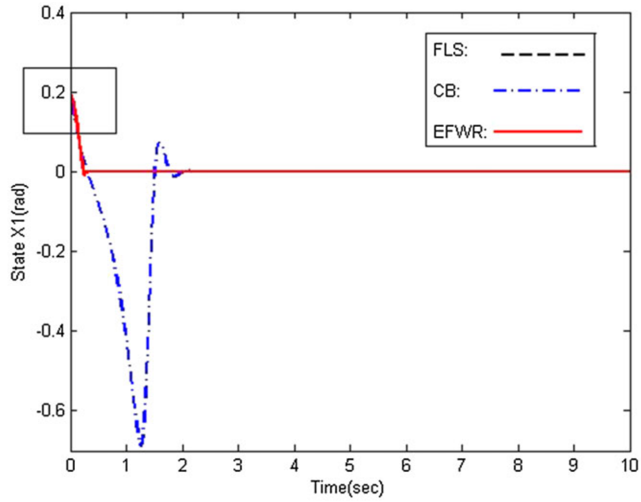

Fig. 3. Response of state variable $x_1(x_1(0) = 0.19 \approx 11°)$.

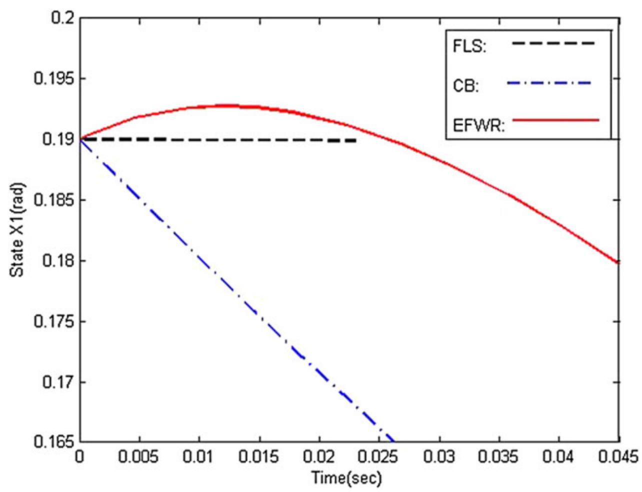

Fig. 4. Locally-enlarged picture of Fig. 3.

Fig. 7 shows the angular response by extended FWR in this paper and the work in [5]. It can be seen that the proposed adaptive controller in this paper remains stable for the large initial condition, while the controller in [5] is out of the work. The updated law is shown in Fig. 8. The estimate values of approximate accuracies and Lipschitz constants, shown in Fig. 9, also tend to zero.

Remark 6. The above simulation is compared with Wang's work [2] and Chen's method [5]. Wang and Chen employed a fuzzy adaptive approach, which contains 25 adaptive laws and one adaptive law, respectively. Wang's work uses the traditional fuzzy adaptive method. It is easy to create the problem of exponential growth in the number of fuzzy rules for the multivariable nonlinear systems. Note that Wang's method uses the smallest initial angle. When the initial angle

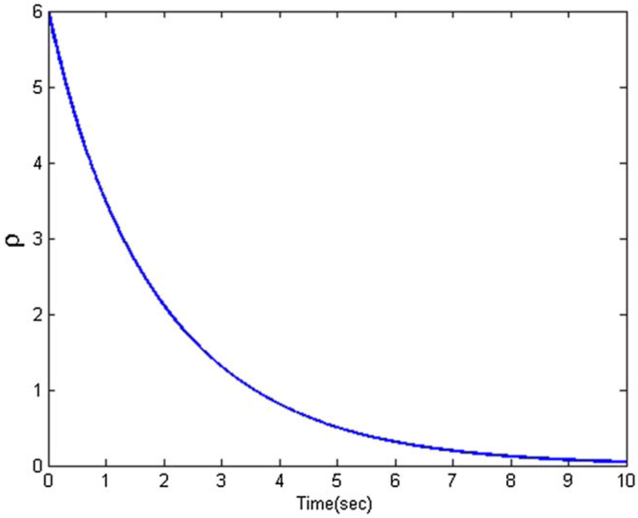

Fig. 5. Response of updated law ρ ($x_1(0) = 0.19 \approx 11°$).

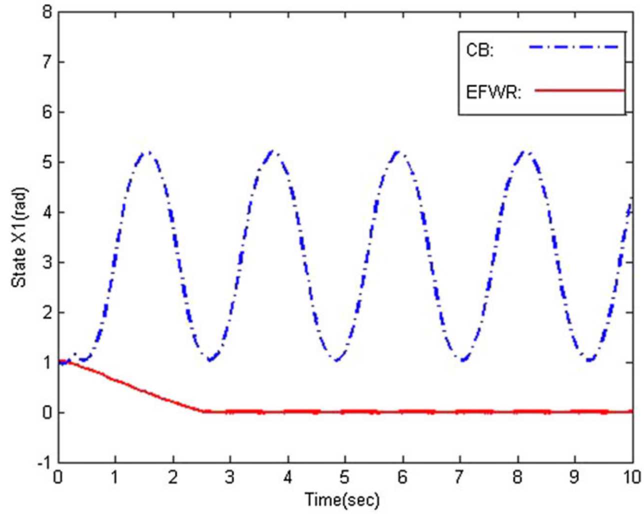

Fig. 7. Response of state variable $x_1(x_1(0) = 1 \approx 57°)$.

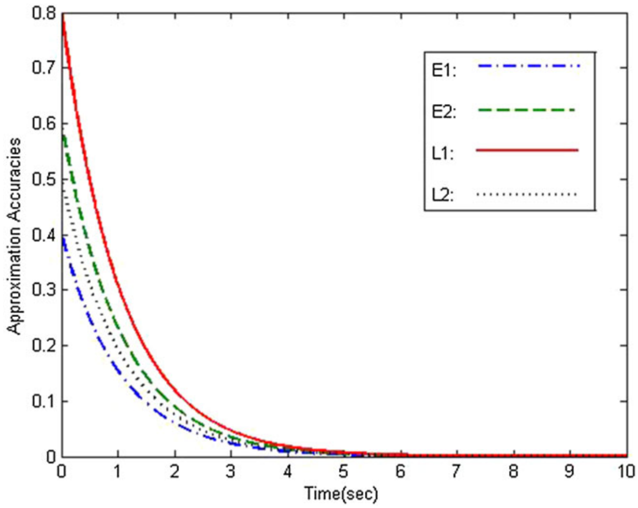

Fig. 6. Response of approximation accuracies and Lipschitz constants ($x_1(0) = 0.19 \approx 11°$).

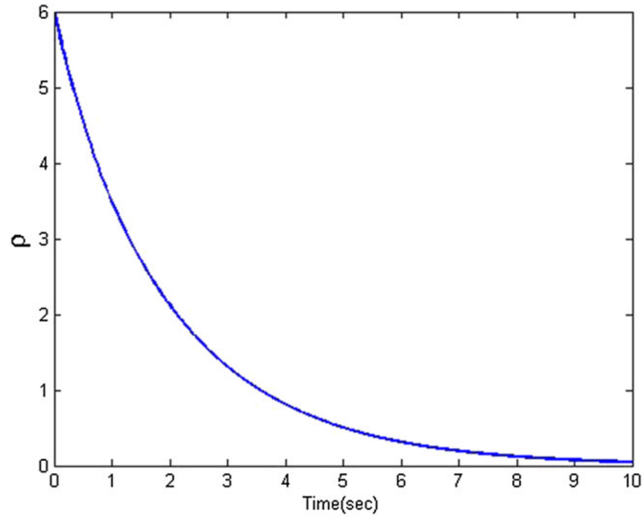

Fig. 8. Response of updated law $\rho(x_1(0) = 1 \approx 57°)$.

becomes bigger, Wang's method ceases to work, but Chen's approach and the method proposed in this paper can still guarantee the system stability. Comparing the number of adaptive laws, one can see that Chen's work uses only one adaptive law. From the analysis of Chen's work design process, we can conclude that the reduction of the number of adaptive laws lies in the cost of bringing in the extra numbers of state variables. In other words, when the number of state variables becomes bigger, the fuzzy basis functions become more and more complex in Chen's work. Therefore, when the initial angle reaches 40^o, Chen's method ceases to work. In this simulation, we employed five adaptive laws to guarantee system stability, including four approximation accuracies and one updated law. From the point view of computational terms, the method proposed in this paper effectively reduces the number of adaptive laws, and deals online with the multivariable-fuzzy adaptive control problem. In that sense, the method proposed in this paper offers an efficient approach for reducing the number of adaptive laws and promoting the stability of nonlinear multivariable systems.

VII. CONCLUSION

A novel adaptive method based on the FWR, for design of control and stability of n-order nonlinear systems, has been proposed in this paper. This method efficiently reduces the number of adaptive laws and relaxes the restriction on universal approximators to be chosen, whose outputs are not

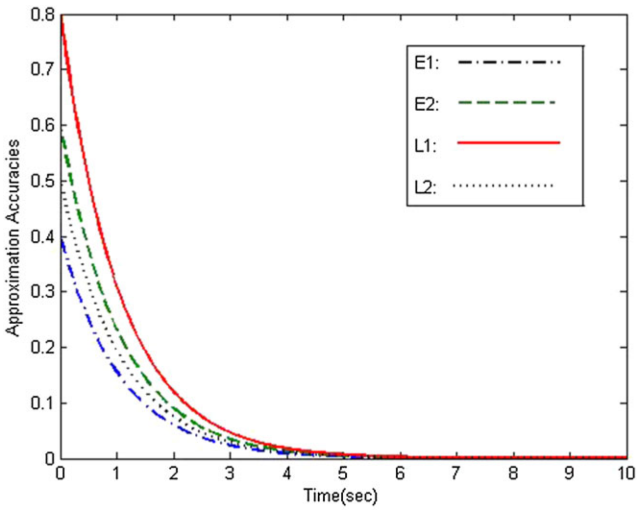

Fig. 9. Response of approximation accuracies and Lipschitz constants ($x_1(0) = 1 \approx 57°$).

requested to be represented as the linear combination of the basis functions. The extended FWR has been proved with good approximation accuracy by tuning the scalar factor. Comparing with the existing results, the main advantage of the approach in this paper is that the FWR can be utilized to design the adaptive controllers for a class of nonlinear uncertain systems. It is easily seen from the above design process that the method in this paper is also suitable for several other different kinds of universal approximators, such as NN, FLS, and partition of unity. This extends the applicability of this approach to many more kinds of practical systems. From the two simulation results in this paper, one can clearly see that the method proposed here gives better results compared with the adaptive fuzzy approach in [2] and [5]. It can be seen in the reduction of the number of "rules", and also in the ability of the designed controllers. Finally, one can conclude that the method proposed in this paper could be a useful control algorithm for stabilization and control of multivariable nonlinear practical systems.

REFERENCES

1. Wang, L. X., "Fuzzy systems are universal approximators," *IEEE Trans. Syst. Man Cybern.,* Vol. 7, No. 10, pp. 1162–1170 (1992).
2. Wang, L. X., "Stable adaptive fuzzy control of nonlinear systems," *IEEE Trans. Fuzzy Syst.,* Vol. 1, No. 1, pp. 146–155 (1993).
3. Tong, S. C. and H. X. Li, "Fuzzy adaptive sliding-mode control for MIMO nonlinear system," *IEEE Trans. Fuzzy Syst.,* Vol. 11, No. 3, pp. 354–360 (2003).
4. Zhang, H. G., J. Yang, and C. Y. Su, "T-S Fuzzy-Model-Based Robust H8 Design for Networked Control Systems With Uncertainties," *IEEE Trans. Ind. Inform.,* Vol. 3, No. 4, pp. 289–301 (2007).
5. Chen, B., X. P. Liu, K. F. Liu, and C. Lin, "Direct adaptive fuzzy control of nonlinear strict feedback systems," *Automatica*, Vol. 45, pp. 1530–1535 (2009).
6. Wang, M., B. Chen, and S. L. Dai, "Direct Adaptive Fuzzy Tracking Control for a Class of Perturbed Strict-Feedback Nonlinear Systems," *Fuzzy Sets Syst.*, Vol. 158, No. 24, pp. 2655–2670 (2007).
7. Wang, M., B. Chen, X. P. Liu, and P. Shi, "Adaptive Fuzzy Tracking Control for a Class of Perturbed Strict-Feedback Nonlinear Time-Delay Systems," *Fuzzy Sets Syst.*, Vol. 159, No. 8, pp. 949–967 (2008).
8. Zhao, Q. C. and Y. Lin, "Adaptive fuzzy dynamic surface control with prespecified tracking performance for a class of nonlinear systems," *Asian J. Control,* Vol. 13, No. 6, pp. 1082–1091 (2011).
9. Liu, Y. J., S. C. Tong, and T. S. Li, "Adaptive fuzzy controller design with observer for a class of uncertain nonlinear MIMO systems," *Asian J. Control,* Vol. 13, No. 6, pp. 868–877 (2011).
10. Moradi, M., M. H. Kazemi, and E. Ershadi, "Direct adaptive fuzzy control with membership function tuning," *Asian J. Control,* Vol. 14, No. 3, pp. 726–735 (2012).
11. Tong, S. C., N. Sheng, and Y. M. Li, "Adaptive Fuzzy Control for Nonlinear Time-Delay Systems with Dynamical Uncertainties," *Asian J. Control,* Vol. 14, No. 6, pp. 1589–1598 (2012).
12. Al-Hadithi, B. M., A. Jiménez, and F. Matía, "Variable Structure Control with Chattering Reduction of a Generalized T-S Model," *Asian J. Control*, Vol. 15, No. 1, pp. 155–168 (2013).
13. Zhou, S. M. and J. Q. Gan, "Low-level interpretability and high-level interpretability: a unified view of data-driven interpretable fuzzy system modeling," *Fuzzy Sets Syst.,* Vol. 159, pp. 3091–3131 (2008).
14. Guillaume S., "Designing fuzzy inference systems from data: an interpretability-oriented review," *IEEE Trans. Fuzzy Syst.,* Vol. 9, No. 3, pp. 426–443 (2001).
15. Kasac, J., B. Novakovic, D. Majetic, and D. Brezak, "Parameters Optimization of Analytic Fuzzy Controllers for Robot Manipulators," *Proc. 9th Int. Conf. Computer Aided Optimum Design in Engineering*, Skiathos, Greece, pp. 23–25 (2005).
16. Novakovic, B. M., "Fuzzy Logic Control Synthesis without Any Rule Base," *IEEE Trans. Syst. Man. Cybern. Part B—Cybern.,* Vol. 29, No. 3, pp. 459–466 (1999).
17. Novakovic, B., D. Scap, and D. Novakovic, "An analytic approach to fuzzy robot control synthesis," *Engineering Applications of Artificial Intelligence*, Vol. 13, pp. 71–83 (2000).
18. Yang, Y. S., G. Feng, and J. S. Ren, "A combined backstepping and small-gain approach to robust adaptive

fuzzy control for strict feedback nonlinear systems," *IEEE Trans. Systems Man Cybern. A—Syst. Hum.*, Vol. 34, No. 3, pp. 406–420 (2004).
19. Yang, Y. S. and C. J. Zhou, "Robust adaptive fuzzy tracking control for a class of perturbed strict feedback nonlinear systems via small-gain approach," *Inf. Sci.*, Vol. 170, pp. 211–234 (2005).
20. Yang, Y. S. and C. J. Zhou, "Adaptive fuzzy H8 stabilization for strict feedback canonical nonlinear systems via backstepping and small-gain approach," *IEEE Trans. Fuzzy Syst.,* Vol. 13, No. 1, pp. 104–114 (2005).
21. Luo, L., Y. H. Wang, Y. Q. Fan, and Y. Zhang, "Novel adaptive control design for nonlinear systems with extended partition of unity," *Asian J. Control*, Vol. 15, No. 3, pp. 911–918 (2013).
22. Haitham, H. and B. Stephen, "Analysis of linear systems with saturation using convex optimization," *Proc. the 37th IEEE Conf. Decis. Control*, Tampa, FL, pp. 903–908 (1998).

VIII. APPENDIX A

8.1 Proof of Lemma 2

Let $S=\|z\|^2-\rho^2\alpha^2+\frac{1}{2}\beta_1^{-1}(\tilde{E}_1^{\mathrm{T}}\tilde{E}_1+\tilde{E}_2^{\mathrm{T}}\tilde{E}_2)+\frac{1}{2}\beta_2^{-1}(\tilde{L}_1^{\mathrm{T}}\tilde{L}_1+\tilde{L}_2^{\mathrm{T}}\tilde{L}_2)$. It is easy to see that the condition $\|z\|>|\rho|\alpha$ means $S>0$. Consider the positive definition function about $V_1=\frac{1}{2}S^2$. The derivative of V_1 about t along the ECS (11) is obtained as follows

$$\begin{aligned}\dot{V}_1&=S\dot{S}\\&=S[\dot{z}^{\mathrm{T}}z-z^{\mathrm{T}}\dot{z}-2\rho\dot{\rho}\alpha^2+\beta_1^{-1}(\tilde{E}_1^{\mathrm{T}}\dot{\hat{E}}_1+\tilde{E}_2^{\mathrm{T}}\dot{\hat{E}}_2)\\&\quad+\beta_2^{-1}(\tilde{L}_1^{\mathrm{T}}\dot{\hat{L}}_1+\tilde{L}_2^{\mathrm{T}}\dot{\hat{L}}_2)]\\&\le S[2\sqrt{n-1}\|z\|^2+2\|z^{\mathrm{T}}B\|(E_1+|EFWR_1|)\\&\quad-2\rho\dot{\rho}\alpha^2+\beta_1^{-1}(\tilde{E}_1^{\mathrm{T}}\dot{\hat{E}}_1+\tilde{E}_2^{\mathrm{T}}\dot{\hat{E}}_2)+\beta_2^{-1}(\tilde{L}_1^{\mathrm{T}}\dot{\hat{L}}_1+\tilde{L}_2^{\mathrm{T}}\dot{\hat{L}}_2)]\\&=-\lambda S.\end{aligned}$$

Obviously, it is true that $\{\mathbb{Z}|S=0\}\subseteq D$. This completes the proof of Lemma 2.

IX. APPENDIX B

9.1 Proof of Lemma 3

Description of the closed-loop system, composed of the controller (18), updated law (19), adaptive laws (20) and the system (12), is given by the relation:

$$\begin{aligned}\dot{z}&=Az+B[f+gu]\\&=(A+BK)z+Bg[-g^{-1}Kz+u_1+g^{-1}f+u_2]\end{aligned}$$

Consider the positive definite function:

$$V_2=z^{\mathrm{T}}Pz+\frac{1}{2\gamma}\rho^2+\frac{1}{2\delta_1}(\tilde{E}_1^{\mathrm{T}}\tilde{E}_1+\tilde{E}_2^{\mathrm{T}}\tilde{E}_2)+\frac{1}{2\delta_2}(\tilde{L}_1^{\mathrm{T}}\tilde{L}_1+\tilde{L}_2^{\mathrm{T}}\tilde{L}_2).$$

If Assumptions 1 and 2 are true, the derivative of $V_2(t)$ along the closed-loop system (12) is given by

$$\begin{aligned}\dot{V}_2\le&-\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}V_2(t)+\frac{\lambda_{\min}(Q)}{2\lambda_{\min}(P)}[\delta_2^{-1}(L_1^2+L_2^2)+\delta_1^{-1}(E_1^2+E_2^2)]\\&+\frac{\rho}{\gamma}\left[\frac{\lambda_{\min}(Q)}{2\lambda_{\min}(P)}\rho+2\gamma\rho\alpha^2\left|1-\frac{1}{\rho}\right|\|PB\|(\hat{L}_1+\hat{L}_2)\right.\\&+\tilde{L}_1\left(\delta_2^{-1}\frac{\lambda_{\min}(Q)}{\lambda_{\min}(P)}\hat{L}_1-2\rho^2\alpha^2\left|1-\frac{1}{\rho}\right|\|PB\|+\delta_2^{-1}\dot{\hat{L}}_1\right)\\&+\tilde{L}_2\left(\delta_2^{-1}\frac{\lambda_{\min}(Q)}{\lambda_{\min}(P)}\hat{L}_2-2\rho^2\alpha^2\left|1-\frac{1}{\rho}\right|\|PB\|+\delta_2^{-1}\dot{\hat{L}}_2\right)\\&+\tilde{E}_1\left(\delta_1^{-1}\frac{\lambda_{\min}(Q)}{\lambda_{\min}(P)}\hat{E}_1-2|\rho|\alpha\|PB\|+\delta_1^{-1}\dot{\hat{E}}_1\right)\\&+\tilde{E}_2\left(\delta_1^{-1}\frac{\lambda_{\min}(Q)}{\lambda_{\min}(P)}\hat{E}_2-2|\rho|\alpha\|PB\|+\delta_1^{-1}\dot{\hat{E}}_2\right)\end{aligned}\tag{C.1}$$

Substituting the updated laws (19) and (20) into (C.1) one obtains that

$$\dot{V}_2\le-\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}V_2(t)+\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}\Theta\tag{C.2}$$

where $\Theta=[\delta_2^{-1}(L_1^2+L_2^2)+\delta_1^{-1}(E_1^2+E_2^2)]$.

Pre and post multiplying (C.2) by $e^{\frac{\chi_{\min}(Q)}{\chi_{\max}(P)}t}$ and then integrating the result from 0 to t show that

$$\begin{aligned}V_2(t)&\le e^{-\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}t}\left[V_2(0)+\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}\Theta\int_0^t e^{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\tau}d\tau\right]\\&\le e^{-\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}t}V_2(0)+\frac{1}{2}\Theta\end{aligned}\tag{C.3}$$

It is concluded from (C.3) that if $t\ge-\frac{\chi_{\max}(P)}{\chi_{\min}(Q)}\ln\frac{\varepsilon}{V_2(0)}$ for a given positive design real ε, then $V_2\le\varepsilon+\frac{1}{2}\Theta$ holds. This means that the state convergent into the neighborhood Ω in finite times, where $\Omega=\left\{(X^{\mathrm{T}}\ \rho\ \tilde{E}_1\ \tilde{E}_2\ \tilde{L}_1\ \tilde{L}_2)|V\le\varepsilon+\frac{1}{2}\Theta\right\}$. Therefore, the following inequalities are obtained: $\|z\|\le\sqrt{\frac{\varepsilon+0.5\Theta}{\lambda_{\min}(P)}}$, $|\rho|\le\sqrt{2\gamma(\varepsilon+0.5\Theta)}$, $\tilde{E}_1^2+\tilde{E}_2^2\le2\delta_1(\varepsilon+0.5\Theta)$, $\tilde{L}_1^2+\tilde{L}_2^2\le2\delta_2(\varepsilon+0.5\Theta)$.

This completes the proof of Lemma 3.

Yinhe Wang received the M.S. degree in mathematics from Sichuan Normal University, Chengdu, China, in 1990, and the Ph.D. degree in control theory and engineering from Northeastern University, Shenyang, China, in 1999. From 2000 to 2002, he was Post-doctor in Department of Automatic control, Northwestern Polytechnic University, Xi'an, China. From 2005 to 2006, he was a visiting scholar at Department of Electrical Engineering, Lakehead University, Canada. He is currently Professor with the Faculty of Automation, Guangdong University of Technology, Guangzhou, China. His research interests include fuzzy adaptive robust control, analysis for nonlinear systems and complex dynamical networks.

Liang Luo received the M.S. and Ph.D. degree in the faculty of applied mathematics and the Faculty of Automation from Guangdong University of Technology, Guangzhou, China, in 2008 and 2011. She is currently a lecturer with college of mathematics and information science, Shaoguan University, Shaoguan, Guangdong, China. Her research interests include nonlinear systems and adaptive robust control.

Branko Novakovic is Professor Emeritus in the Department of Robotics and Automation of Manufacturing Systems at Faculty of Mechanical Engineering and Naval Architecture,University of Zagreb, Croatia. Prof. Novakovic received his Ph.D. in Mechanical Engineering from the University of Zagreb in 1978. His research interests include control systems, robotics, neural networks, and fuzzy control. He is author of two books: Control Systems (1985), and Control Methods in Robotics, Flexible Manufacturing Systems and Processes (1990) and co-author of a book Artificial Neural Networks (1998).

Josip Kasac is Associate Professor in the Department of Robotics and Automation of Manufacturing Systems at Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Croatia. Dr. Kasac received his Ph.D. in Mechanical Engineering from the University of Zagreb in 2005. His research interests include control of nonlinear mechanical systems, repetitive control systems, optimal control and fuzzy control.